Abstract—In this paper, we propose two differential relaying strategies: Combined-Transmit-Relay (C-T-R) and Transmit-Combined-Relay (T-C-R), which could be used by the base-station to transmit data of two users over downlink channels in the two-user cooperative communication network. Differential schemes are used so the users do not require the knowledge of channel gains for decoding of their data. The performance of both the schemes is evaluated for decode-and-forward, selection relaying, and incremental relaying protocols. In the case of decode-and-forward protocol, the performance heavily depends upon the signal to noise ratio of the link between source and relay. We propose an exact probability of error based approach, which can be used to regulate the erroneous relaying of data. Both C-T-R and T-C-R are able to achieve performance gain as compared to the direct transmission differential scheme in the case of selection relaying and incremental relaying protocols at high SNRs. The proposed schemes also perform better than the direct transmission differential scheme.

Index Terms: Differential relaying, Downlink channels, Cooperative network

I. INTRODUCTION

Cooperative communication systems are now receiving more and more attention as they can provide all the advantages of multiple-input multiple-output (MIMO) communications systems like diversity, improved capacity, and coding gain without any physical antenna array [1]. As the cost of installation of a physical antenna array is much higher and many issues are still remaining to be solved, the cooperative MIMO seems to be a promising technique, which shares the resources between all the users. A lots of research has been performed over the cooperative MIMO networks [1], [2], [3] and protocols like amplify-and-forward, decode-and-forward, coded cooperation, selection relaying, and incremental relaying are studied and compared in detail in [2]. All the protocols, except decode-and-forward, are proved to be useful for increasing the spatial diversity of the cooperative MIMO system [2].

Most of the cooperative communication study performed so far assumes perfect channel knowledge at the user and base-station (BS) side. However, the training based method used for estimation of channel gains reduces the effective data rate and also it is hard to acquire the accurate knowledge about the channel specially in fast fading channels, which remain stationary for small durations. Therefore, these relaying protocols will not work well in such cases and the spacial diversity cannot be achieved in cooperative MIMO networks.

Hence, we may use differential modulation to avoid the need of channel knowledge by the receiver for decoding of data. The differential modulation for two-user cooperative diversity system is proposed in [5],[6]. However, the method of [5] is applicable only for uplink channels in the cooperative network. It is very intuitive that it is more difficult for a mobile user to acquire the accurate channel knowledge as compared to the base station (BS), which is far larger in size, is located in one place to implement many power consuming signal processing algorithms for estimation of the channel gains.

Therefore, a differential downlink strategy can reduce much more load of the channel estimation from the mobile users as compared to the uplink differential strategy and can save the precious battery and help in reducing the weight of a mobile handset. A differential transmission strategy for amplify-and-forward downlink cooperative communication system is proposed in [6]. But this scheme depends upon the optimum power allocation, which depends upon the channel and noise statistics. When the power allocation is non-optimal, the performance of [6] degrades. In addition, the decoding process in [6] is still depends upon the channel knowledge and the users need to update its knowledge about the channel gains and noise statistics constantly.

In this paper, our main contribution is to propose two differential modulation strategies for two-user cooperative diversity network, which are applicable over downlink channels. These schemes avoid the need of channel gain estimation at the mobile station, for decoding of data. These schemes also perform better than the direct DPSK (no cooperation). However, the performance of [6] degrades. In addition, the decoding process in [6] is still depends upon the channel knowledge and the users need to update its knowledge about the channel gains and noise statistics constantly.

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The rest of this paper is organized as follows: In Section II, the system model is given, differential coding for SISO system is explained, and C-T-R and T-C-R relay strategies are discussed in detail. Performance of the proposed differential relaying schemes with the decode-and-forward protocol is discussed in Section III. In Section IV, criterion for selection of relayed data is proposed. Performance of the proposed...
schemes in association with the proposed criterion is discussed in Section V. Section VI contains some conclusions.

II. SYSTEM MODEL

We consider a cellular system in which the base station (BS) is transmitting information of two cooperative users over downlink channels. The source, relay, and destination are equipped with single antenna and can only receive or transmit at a time. We have assumed that all interuser channels, BS-relay, and BS-user channels are independent of each other. All channels are frequency flat and quasi static, i.e., they are fixed for multiple time intervals. In addition, time division multiple access (TDMA) is used for transmission and relaying of information symbols. We have considered two different scenarios of cooperative communication between the users. In the first scenario, shown in Fig. 1(a) the interuser channels are good and the users may work as relay for each other. In the second scenario, shown in Fig. 1(b) we also have a relay selected by both the users/BS.

![Fig. 1. Two-user cooperative communication system (a) without relay (b) with a relay.](image)


In this section, we review the differential modulation for SISO systems. Let $\Psi$ be an $M$-PSK constellation and $b[n] \in \Psi$ be the data to be transmitted over a flat fading, quasistatic channel, which remains stationary for at least two time intervals. In differential encoding, we transmit following symbol:

$$s[n] = b[n] s[n-1].$$  \hspace{1cm} (1)

The differential transmission begins with $s[0]$ which is normally set to unity. If $r[n-1]$ and $r[n]$ are the data samples received in two consecutive time intervals, then the estimate of $b[n]$ are obtained as follows:

$$\hat{b}[n] = \arg \max_{b \in \Psi} \{ r[n] r^* [n-1] b \}. \hspace{1cm} (2)$$

B. Combined-Transmit-Relay Strategy for Two-User Cooperative Communication System

In this scheme, we consider the cooperative communication system shown in Fig. 1(a). The BS have to transmit $b_1[n]$ and $b_2[n] \in$ BPSK, $0 \leq n \leq N/3 - 1$ to User 1 and User 2, respectively. We consider a TDMA transmission scheme, where $N/3$ time slots are devoted to each transmission and the channel is assumed to be fixed over these $N/3$ time intervals, where $N$ is the length of one frame. The BS first encodes $b_1[n]$ and $b_2[n]$ into differential symbols $s_1[n]$ and $s_2[n]$, $0 \leq n \leq N/3 - 1$, respectively, before transmission. In the first $N/3$ time intervals, the BS transmits a combined version of $s_1[n]$ and $s_2[n]$ in which, $s_1[n]$ is kept on the I-axis and $s_2[n]$ is kept on the Q-axis. In the next $N/3$ time intervals, the BS transmits $s_1[n]$ and User 2 relays the estimated $\hat{s}_1[n]$, to User 1 and in the further $N/3$ time intervals BS and User 1 transmit $s_2[n]$ and $\hat{s}_2[n]$, respectively, to User 2. The transmission schedule of this scheme is shown in Table 1. The received signal in the first $N/3$ time intervals corresponding to the direct transmission is

$$r_{s_1}[n] = h_{s_1} (s_1[n] + j s_2[n]) + e_{s_1}[n], \hspace{1cm} 0 \leq n \leq N/3 - 1,$$

where $h_{s_1}$ is the channel between BS and User 1 and $e_{s_1}[n]$ is additive white Gaussian noise received by User 1. The data received at the User 2 in the first $N/3$ time intervals at any $0 \leq n \leq N/3 - 1$ will be

$$r_{s_2}[n] = h_{s_2} (s_1[n] + j s_2[n]) + e_{s_2}[n].$$  \hspace{1cm} (4)

In the next $N/3$ time intervals, the received data at User 1 at $N/3 \leq n \leq 2N/3 - 1$ is

$$r_{s_{21}}[n] = h_{s_{21}} s_1[n-N/3] - j h_{s_{21}} s_1^*[n-N/3] + e_{s_{21}}[n], \hspace{1cm} (5)$$

where $s_1[n-N/3]$ is the differential symbol transmitted by User 2 corresponding to the estimate of the User 1’s data obtained by it at $0 \leq n \leq N/3 - 1$ and $h_{s_{21}}$ is the channel between User 1 and User 2. The additional multiplication of $j$ and conjugation in (5) is done to maintain the normal differential decoding at the receiver. The receiver at User 1 computes the following for decision variable:

$$d_{c}[n] = (r_{s_{21}}[n] r_{s_{21}}^* [n-1] + (r_{s_1}[n-N/3] \times r_{s_1}^*[n-N/3-1]), \hspace{1cm} N/3 \leq n \leq 2N/3 - 1.$$  \hspace{1cm} (6)

It can be shown that the real part of $d_{c}[n]$ is sufficient to decode the data of User 1. Therefore, for deciding about the BPSK symbol the receiver only checks the sign of the real part of $d_{c}[n]$. The similar process is followed by User 2 to decode its data.

C. Transmit-Combined-Relay Strategy for Two-User Cooperative Communication System

This scheme is applicable to the case when there is a cooperative communication as shown in Fig. 1(b). In this case, the users and BS select one relay which has healthy channels to receive information from BS and relay information to users. Assume $s_1[n]$ and $s_2[n]$ are differential symbols of User 1

<table>
<thead>
<tr>
<th>Transmission</th>
<th>Time-Time Interval</th>
<th>Second-Time Interval</th>
<th>First-Time Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS-U1</td>
<td>$s_1[n] + s_2[n]$</td>
<td>$s_1[n] + s_2[n]$</td>
<td>$s_1[n] + s_2[n]$</td>
</tr>
<tr>
<td>BS-U2</td>
<td>$s_1[n] + s_2[n]$</td>
<td>$s_1[n] + s_2[n]$</td>
<td>$s_1[n] + s_2[n]$</td>
</tr>
<tr>
<td>User-U1</td>
<td>$s_1[n] + s_2[n]$</td>
<td>$s_1[n] + s_2[n]$</td>
<td>$s_1[n] + s_2[n]$</td>
</tr>
</tbody>
</table>
and User 2, respectively. We also assume that the data of both users belongs to BPSK constellation only. In this scheme, the BS transmits the symbols of User 1 in the first \( N/3 \) time intervals and the symbols of User 2 in the next \( N/3 \) time intervals. In the final \( N/3 \) time intervals of the frame, only the relay transmits a combined version of the data of both the users. The transmission schedule of this scheme is shown in Table 2. The data received by User 1 in the first \( N/3 \) time intervals is

\[
r_{s1}[n] = h_{s1}s_1[n] + e_{s1}[n], \quad 0 \leq n \leq N/3 - 1. \tag{7}
\]

In the same time intervals the data received by the relay is

\[
r_{s2}[n] = h_{s2}s_2[n] + e_{s2}[n], \quad 0 \leq n \leq N/3 - 1. \tag{8}
\]

User 2 and relay receive the following data the next \( N/3 \) time intervals:

\[
r_{s2}[n] = h_{s2}s_2[n] + e_{s2}[n], \quad N/3 \leq n \leq 2N/3 - 1, \tag{9}
\]

\[
r_{s2}[n] = h_{s2}s_2[n] + e_{s2}[n], \quad N/3 \leq n \leq 2N/3 - 1. \tag{10}
\]

The relay first computes the following for the decision about the data of User 1:

\[
dr_{1}[n] = r_{s1}[n]r_{s1}^{*}[n - 1], \quad 0 \leq n \leq N/3 - 1, \tag{11}
\]

and decides about the BPSK data by comparing the real part of \( dr_{1}[n] \) with zero. Similarly, it decodes the data of the second user. The relay encodes the estimated symbols of User 1 and User 2 into differential form and makes a combined version of these symbol and retransmits this new symbol to the users in the final \( N/3 \) time intervals. The received data at User 1 is

\[
r_{c}[n + 2N/3] = h_{c}(\hat{s}[n] + j\hat{s}[n + N/3])
\]

\[+ e_{c}[n + 2N/3], \quad 0 \leq n \leq N/3, \tag{12}
\]

where \( \hat{s}[n] \) and \( \hat{s}[n + N/3] \) are the differential symbols transmitted by relay corresponding to the estimates of data of User 1 and User 2. The receiver at User 1 computes the following before decision:

\[
dc[n + 2N/3] = r_{s1}[n]r_{s1}^{*}[n - 1]
\]

\[+ r_{c}[n + 2N/3]r_{c}^{*}[n + 2N/3 - 1], 0 \leq n \leq N/3. \tag{13}
\]

It can be shown that the real part of \( dc[n + 2N/3] \) is sufficient to decode the data of User 1. Similarly, User 2 can also decode its data.

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**TABLE II**

<table>
<thead>
<tr>
<th>Transmission Schedule for T-C-R Differential Relaying.</th>
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<tbody>
<tr>
<td>Time Period</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>BS to User 1</td>
</tr>
<tr>
<td>BS to User 2</td>
</tr>
<tr>
<td>BS to User 2</td>
</tr>
</tbody>
</table>

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**Fig. 2.** Performance of C-T-R scheme with decode-and-forward relaying.

**III. PERFORMANCE OF THE PROPOSED DIFFERENTIAL RELAYING SCHEMES WITH DECODE-AND-FORWARD PROTOCOL**

In the decode-and-forward protocol, the relay decodes the received data before retransmission [2]. The performance of the C-T-R and T-C-R schemes with decode-and-forward relaying is shown in Fig. 2 and Fig. 3, respectively. It is seen that the performance of the proposed schemes heavily depends upon the signal-to-noise-ratio (SNR) of the link between BS and relay (relaying user in C-T-R). If the SNR of the link between the source and the relay is very good, the relay can decode the data successfully, otherwise, it may forward the erroneous symbols to the destination. Both scheme are seen to achieve the performance gain in the case, when relay always transmits the correct symbol to the destination. However, it is an unrealistic scenario. The performance of the proposed schemes can be improved if the relay can decide about the quality of reception before retransmitting the data (selection relaying [2]). Moreover, the destination can also decide about the quality of reception and send feedback to the relay about it (incremental relaying [2]).

**IV. CRITERION FOR SELECTION OF RELAYED DATA**

In the selection relaying protocol proposed in [2], transmission or non-transmission of signal from relay depends upon the condition of the channel. In the our differential relaying schemes, the source or destination does not have knowledge about the channel gains. Therefore, we propose a new criterion, which is used by the relay to determine the quality of reception of the data. This criterion is based on a simple method of channel power estimation and calculation of the exact probability of error in making the decision about the differentially encoded data. Let the channel remain stationary...
over $N/3$ time intervals, i.e., it is a flat fading channel. As source, destination, and relay works in TDMA and as shown in Tables 1 and 2, the relay/relaying user will receive the data from BS over at least $N/3$ time intervals in both strategies. Let $r = hs + e$ be the vector of data received over the $N/3$ time intervals by the relay, where $s = [s_0, s_1, ..., s_{N/3-1}]$ denotes the data vector, $h$ is the channel gain between BS and relay, and $e = [e_0, e_1, ..., e_{N/3-1}]$ denotes the noise vector. Then $\mathbb{E} \{ rr^H \} = N/3 \| h \|^2 + N/3 \cdot \sigma^2_e$, where $\sigma^2_e$ is the variance of noise. Hence, an estimator of channel power $p_h = \mathbb{E} \{ |h|^2 \}$ is as follows [4]:

$$p_h = \frac{\mathbb{E} \{ rr^H \} - (N/3) \sigma^2_e}{N/3} \approx \frac{rr^H}{N/3}. \quad (14)$$

As the noise power is unity we actually obtain the SNR between the BS and relay from (14). The probability of error in the received data is always a function of SNR of the link. The exact expression of probability of error for Binary differential PSK is given in [7] as

$$P \{ E \} = \frac{1}{2} \exp (-\gamma), \quad (15)$$

where $\gamma$ is the SNR of the link. The closed form expression of the exact probability of error in four phase differential PSK is given in [7] as

$$P \{ E \} = Q_1(a, b) - \frac{1}{2} I_0(ab) \exp \left[ -\frac{1}{2} (a + b) \right], \quad (16)$$

where $Q_1(a, b)$ is the Marcum Q function defined as

$$Q_1(a, b) = \exp \left\{ \frac{1}{2} (a^2 + b^2) \right\} \sum_{k=0}^{\infty} \left( \frac{a}{b} \right)^k I_k(ab), \quad (17)$$

where $b > a > 0$ and $I_k$ is the modified Bessel function, defined as

$$I_k(x) = \sum_{m=0}^{\infty} \frac{(x/2)^{k+2m}}{m! \Gamma(k + m + 1)}, \quad x \geq 0, \quad (18)$$

and $a = \sqrt{\gamma (2 - \sqrt{2})}$ and $b = \sqrt{\gamma (2 + \sqrt{2})}$. The relay terminal first estimates the SNR of the link between the BS and relay and then calculates the exact probability of error from (15) and (16). The average probability of error over the link is set as the threshold. At each time, the relay/relaying user calculates the exact probability of error in the received data and if it is greater than the average probability of error it stops the transmission, and if it is less than or equal to the average probability of error it relays the data to source. Similarly, the destination can also decides about whether it needs relaying of data or not after checking the quality of the received data using the proposed criterion, in incremental relaying protocol [2].

V. PERFORMANCE OF C-T-R AND T-C-R SCHEMES WITH THE PROPOSED CRITERION

We perform simulations for the two user cooperative diversity system with the C-T-R and T-C-R differential relaying strategies. The channel between BS and the users and between the relay and users are assumed to be circular complex Gaussian. The signal power transmitted in one time interval is kept as unity. The SNR is varied by varying the variance of the channel coefficients. The noise is assumed to be complex valued additive white Gaussian (AWGN) with unity variance.

Figs. 4 and 6 show the performance of C-T-R and T-C-R relay strategies, respectively, when the criterion proposed in Section V is applied at the relaying terminal. In this case, the
relay decides about the channel through the proposed criterion and blocks the transmission of the symbols detected with error (selection relaying). It is seen that the performance of both schemes improves very much with the selection relaying. Both of them perform very close to the perfect relay knowledge case, with high (35 dB) SNR. The results in Figs. 4 and 6 demonstrate that the selection criterion proposed in Section V retrieves the diversity of the cooperative communication system.

The simulation are also performed, for incremental relaying protocol. In this case, the destination user makes use of the criterion given in Section IV and decides whether it could decode the data with permissible error or not. If it finds the channel good, it signals the relay not to relay the data, with one feedback bit and if the channel is bad it requests the relay to transmit the data. It can be observed from Figs. 5 and 7 that the system works very close to the perfect knowledge at the relay case at high SNRs, with incremental relaying also, in both C-T-R and T-C-R relaying schemes.

VI. CONCLUSIONS

We have proposed two differential relaying schemes for downlink channels, which obviate the need of channel gain estimation for the decoding of data in a cooperative diversity network. We have also proposed an exact probability of error based criterion for avoiding erroneous relaying of data from the relay terminal. Both schemes are able to achieve performance gain in association with the proposed criterion. The proposed schemes also perform better than the direct transmission differential scheme.

REFERENCES